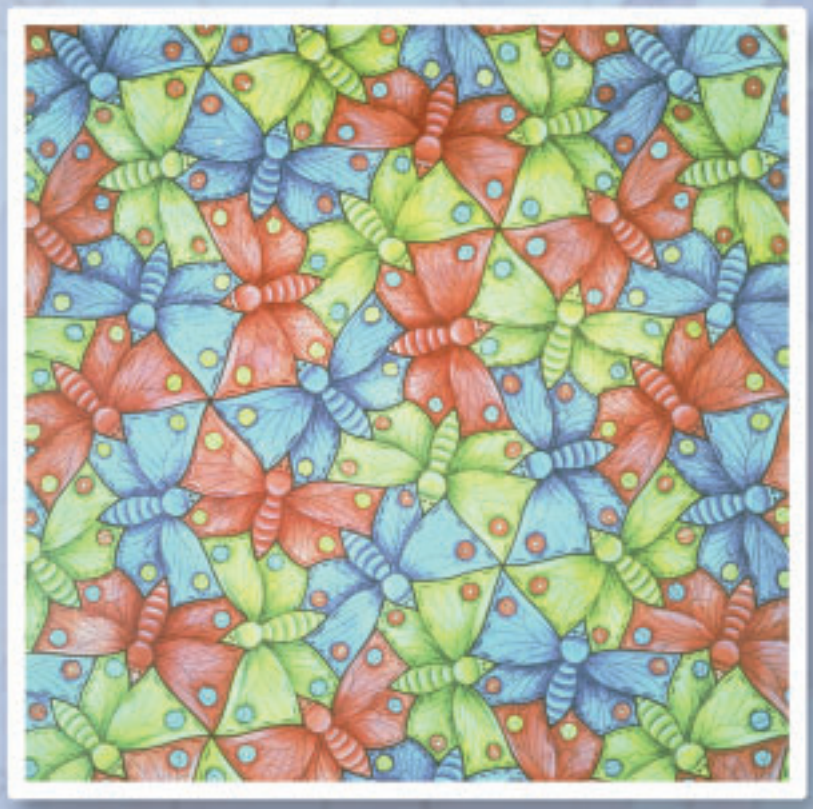
UNIT 7

Transformational Geometry in Art



M.C. Escher was a Dutch graphic artist.

He made many drawings like these.

Learning Goals

- describe position on a grid
- apply translations, reflections, and rotations
- pose and solve problems with transformations
- identify congruent figures
- construct figures with one line of symmetry
- explore tiling patterns and tessellations

Geometry

Key Words

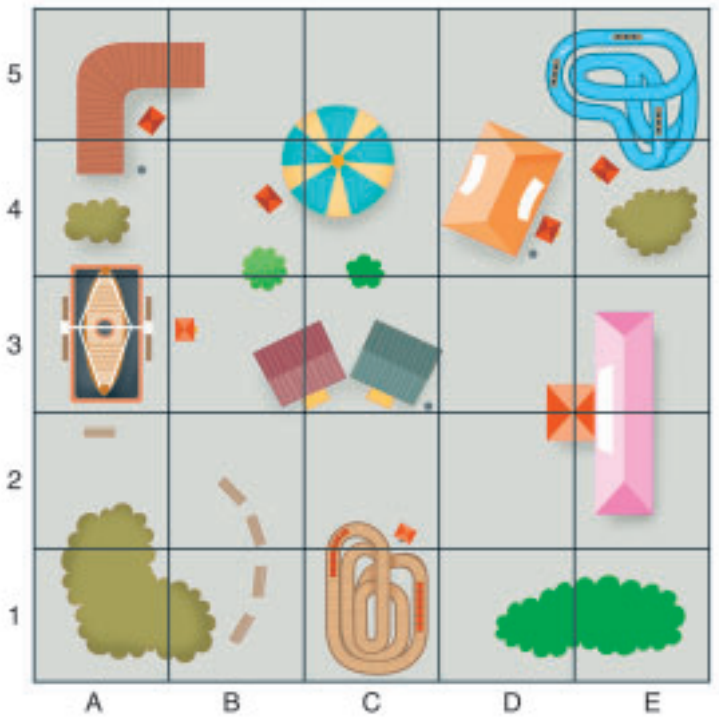
- coordinate systems
- symmetrical
- tiling pattern
- tessellation
- tessellate

- Describe the figures you see.
- Which figures have been translated? Reflected? Rotated?

LESSON 1

Coordinate Systems

Here is a map of an amusement park.
The roller coaster is at C1.
What are the coordinates of the water ride?
The swinging ship?



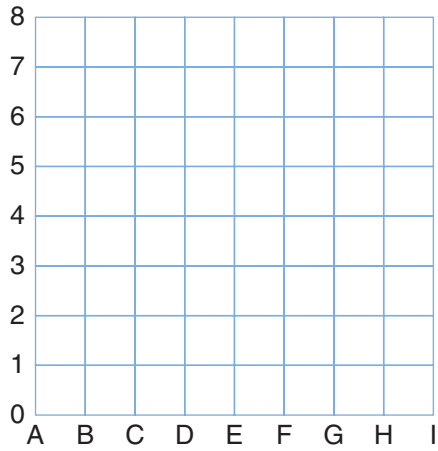
Explore

Each of you will need grid paper and a ruler.

➤ Copy this grid.
Mark a point at each pair of coordinates.
Use a ruler to connect the points in the order you draw them:

- B1 to A3
- A3 to D6
- D6 to G3
- G3 to F1
- F1 to B1

Which polygon did you draw?



➤ Draw a polygon on a grid.
The vertices must be where the grid lines cross.
List the coordinates of the vertices.
Trade lists of coordinates with your partner.
Draw your partner's polygon.
How does the polygon you drew compare with your partner's?

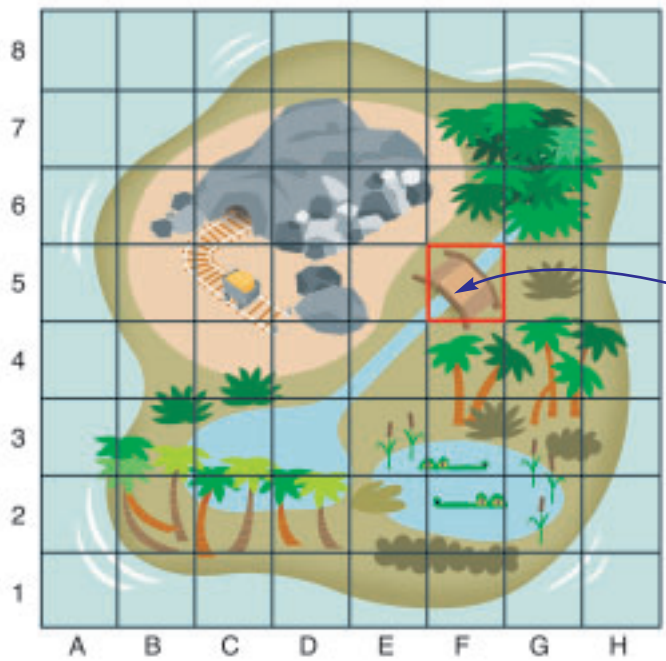
Show *and* Share

Share your work with another pair of students.
How did you know the coordinates of each vertex?
How are these coordinates different from the coordinates on the map of the amusement park?

Connect

Here are 2 kinds of **coordinate systems**.

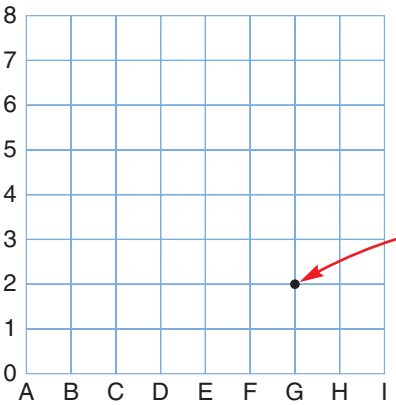
➤ On a map, a pair of coordinates refers to a square.



F5 is the square outlined in red.

The bridge is in F5.

➤ On a grid, a pair of coordinates refers to a point where the grid lines cross.

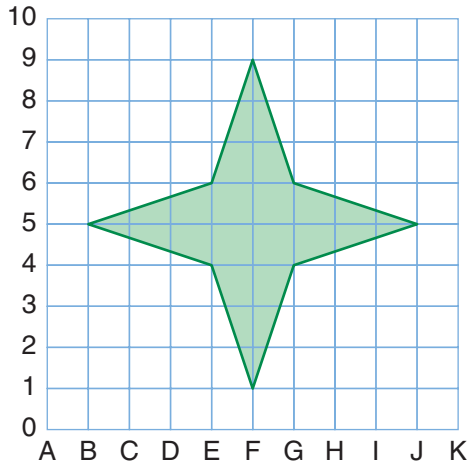


G2 is the point where the grid lines labelled "G" and "2" cross.

The dot is at G2.

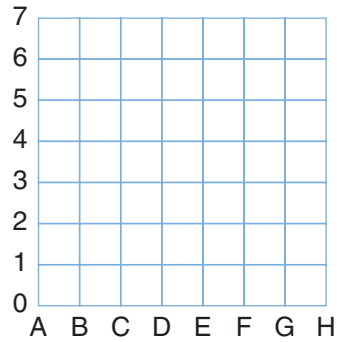
Practice

1. Write the coordinates of each vertex.



2. Use grid paper. Copy this grid.
Mark a point at each pair of coordinates.
Connect each pair of points in order.
What is the message?

- B6, B2
- D6, D2
- B4, D4
- F6, F2
- E6, G6
- E2, G2



3. You will need grid paper. Use question 2 as a model.
Choose a 2-digit number. Draw it on a grid.
Write instructions so someone else can draw your number.
Include coordinates.
Trade instructions with a classmate.
Find your classmate's number.

Reflect

Describe 2 kinds of coordinate systems.
When is each system used?
Use pictures and words to explain.

Numbers Every Day

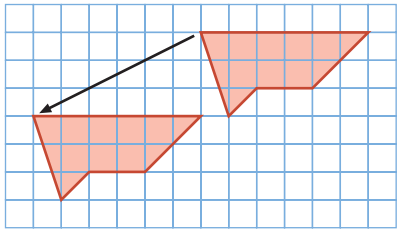
Calculator Skills

Find 3 consecutive even numbers that have a sum of 42. Find the product of the 3 numbers.

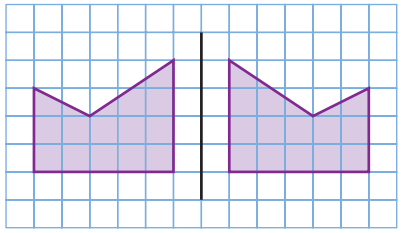
LESSON 2

Transformations

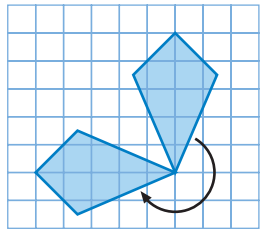
A translation, a reflection, and a rotation are transformations.



a translation



a reflection



a rotation

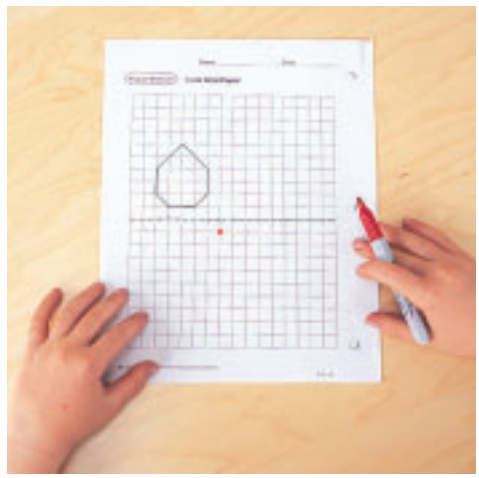
Explore

You will need a copy of this figure, a ruler, a Mira, grid paper, and tracing paper.

Work together.

- Draw and label the image of the hexagon after each transformation:
  - a translation of 2 squares right, 3 squares up
  - a reflection in the broken line
  - a $\frac{1}{2}$ turn about the dot
- Compare the hexagon and its images. Look at size, shape, and the way each one faces.
- Predict the image after each transformation:
  - a $\frac{1}{4}$ turn clockwise about the dot
  - a translation of 1 square left, 7 squares down

  Check your predictions.

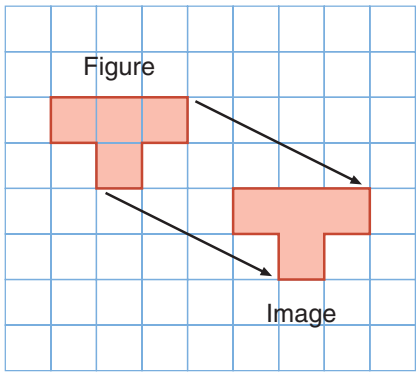


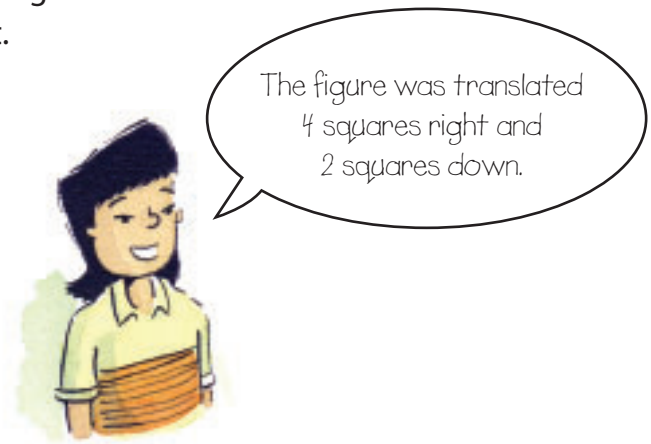
Show *and* Share

Share your work with another pair of students.
How is each image different from the figure?
How are a figure and its image the same?

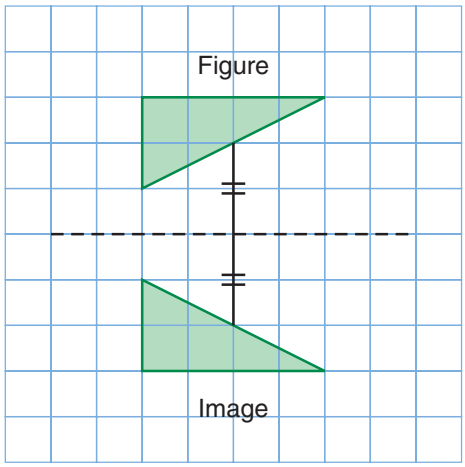
Connect

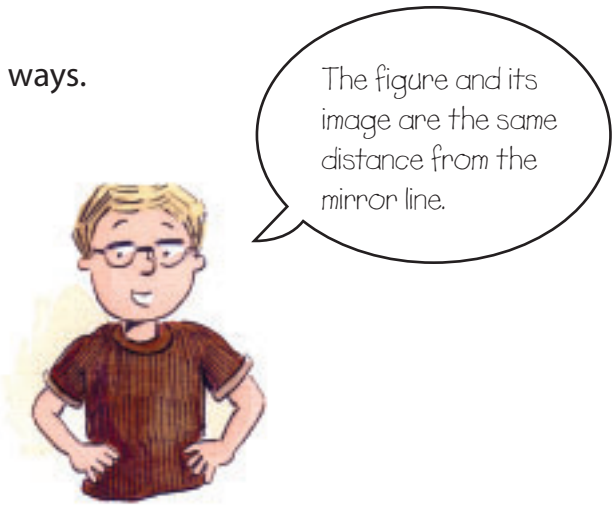
➤ A translation moves a figure along a straight line.
The figure and its image are congruent.



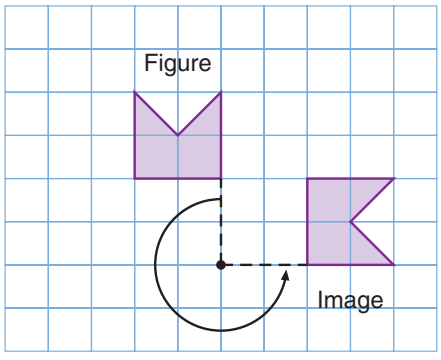


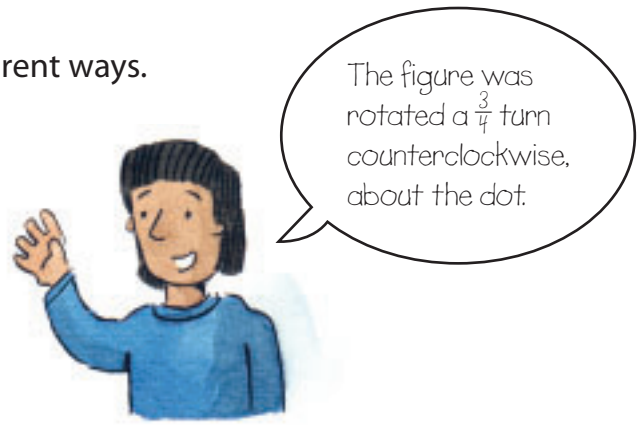
➤ A reflection is a flip.
The figure and its image face opposite ways.





➤ A rotation is a turn.
The figure and its image may face different ways.

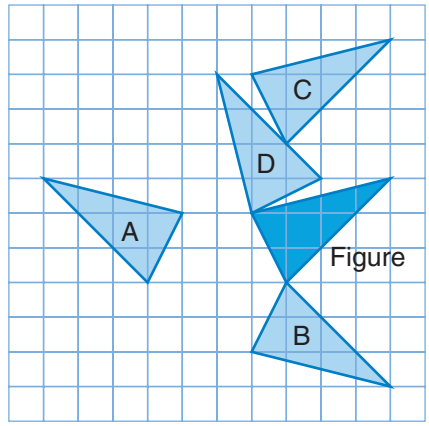




When you translate, reflect, or rotate a figure,
the figure and its image are congruent.

Practice

Use tracing paper when it helps.



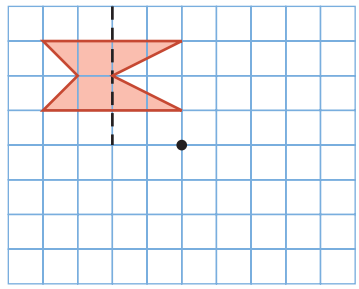
1. Describe the transformation that moves the figure to each image.
Can you describe any movements in more than one way? Explain.

a) Image A

b) Image B

c) Image C

d) Image D



2. Copy this figure on 1-cm grid paper.
Draw the image after each transformation.

a) a translation of 4 squares down and 1 square right

b) a reflection in the broken line

c) a $\frac{1}{4}$ turn counterclockwise about the dot

3. Use 1-cm grid paper.

a) Draw a square.

- Choose a translation.
- Choose a mirror line.
  Reflect the square in the mirror line.
- Choose a turn centre, a fraction of a turn, and the direction.
  Rotate the square about the turn centre.

Draw and label the image of the square after each transformation.
Compare the square to each image.

b) Repeat part a.
This time start with a rectangle.
Compare the rectangle to each image.

c) When you see a square and its image, can you always tell what the transformation was? Explain.
Answer the same question for a rectangle and its image.

Numbers Every Day

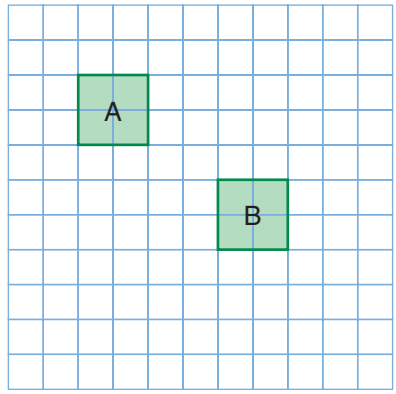
Number Strategies

Find the mean of the numbers in each set.

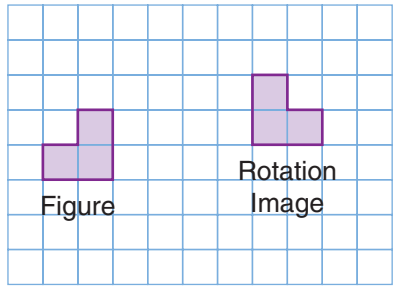
- 15, 25, 5, 10, 45
- 30, 10, 60, 20, 40, 20
- 3, 8, 9, 12, 6, 5, 6

4. **a)** Use a transformation.
How many different ways can you move Figure A to coincide with Figure B?
Describe each transformation.

b) Use two transformations.
How many different ways can you move Figure A to coincide with Figure B?
Describe each pair of transformations.



5. Copy this hexagon and its final image on grid paper.
The hexagon was reflected.
Then its reflection image was rotated.
Where could the reflection image be?
Explain.
Show your work.



6. Use question 5 as a model.
Create your own problem that can be solved using transformations.
Trade problems with a classmate.
Solve your classmate's problem.

Reflect

Can the image after a translation be the same as the image after a reflection?
Use pictures and words to explain.

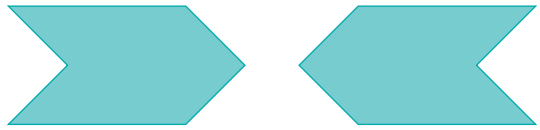
At Home

Look for an example of a transformation.
Which transformation moves the figure to its image?

LESSON 3

Congruent Figures

These hexagons are congruent.
They have the same size and shape.



Explore

You will need a geoboard, geobands, and dot paper.

Use a geoband to make a large square.
Use geobands to divide the square into 2 congruent polygons.
How many different ways can you do this?

Draw each polygon on dot paper.

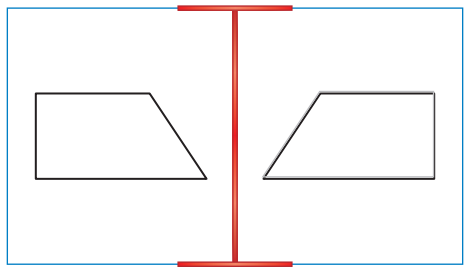
Show *and* Share

Show your work to a classmate.
How did you know the polygons in each pair were congruent?
What strategies did you use to make congruent pairs of polygons?

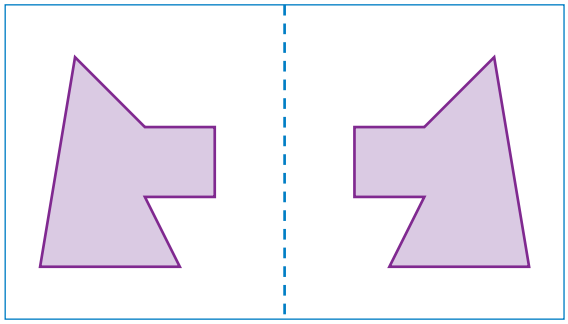
Connect

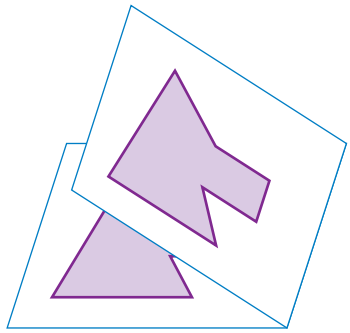
We know that a figure and its transformation image are congruent.
We may use this to check if figures are congruent.

➤ Use a Mira.
The reflection of one trapezoid coincides with the other.
The trapezoids are congruent.



➤ Fold the paper so one polygon coincides with the other.





The fold line is a mirror line.
The polygons coincide.
Each polygon is a reflection of the other.

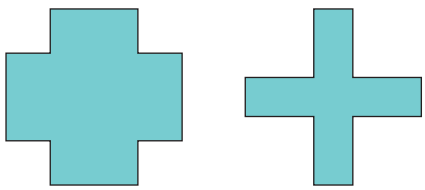
➤ Use tracing paper.

Rotate then translate the tracing to see if it coincides with the other figure.

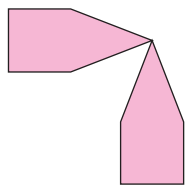
Practice

1. Which pairs of figures are congruent? How do you know?

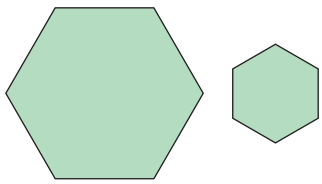
a)



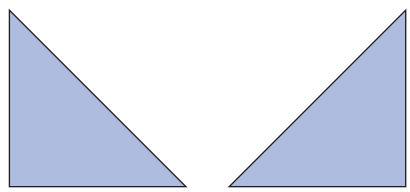
b)



c)



d)



2. Look at the figures in question 1.
Which figures show a transformation? How do you know?
Identify the transformation.

3. Use a geoboard.

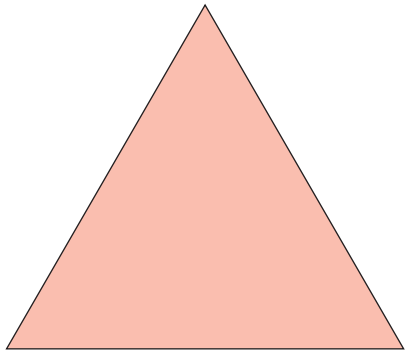
a) Make a polygon. Draw the polygon on dot paper.
Label it A.

b) Make a polygon that is congruent to Polygon A.
Draw the polygon on dot paper. Label it B.
How do you know Polygons A and B are congruent?

c) Make a polygon that is not congruent to Polygons A and B.
Draw the polygon on dot paper. Label it C.
How do you know Polygon C is not congruent to Polygons A and B?

4. Use a 5 by 5 geoboard.
Divide the geoboard into 8 congruent parts.
How many different ways can you do this?
Record each way on dot paper.
How do you know the parts are congruent?

5. Trace this equilateral triangle.
Divide the triangle into 4 congruent triangles.
Use any materials that help.
Explain how you did it.
How do you know the triangles are congruent?



Reflect

How can you demonstrate 2 figures are congruent?
Use pictures and words to explain.

Numbers Every Day

Number Strategies

Find 2 numbers with a difference of 9 and a product of 36.

Using a Computer to Explore Congruent Figures

Work with a partner.

Use *AppleWorks*.
Follow these steps to create polygons and check if they are congruent.

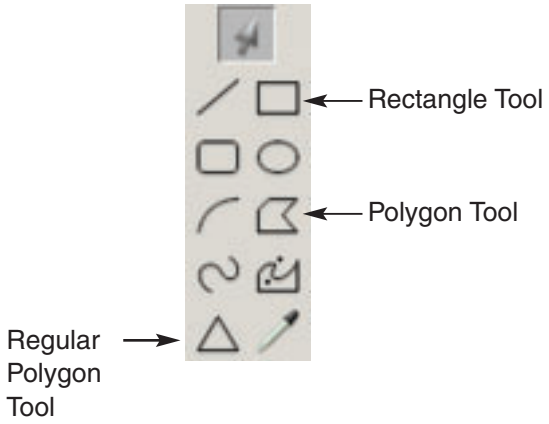
1. Open a new drawing document. Click: 

2. If a grid appears on the screen, go to Step 3.
   If not, click: Options, then click: Show Graphics Grid

3. Check the ruler units are centimetres.
   Click: Format
   then click: Rulers
   then click: Ruler Settings...
   Choose these settings:

   Click: OK

4. Use these tools to draw polygons:



To make a **rectangle**, click
the Rectangle Tool.
Click and hold down
the mouse button.
Drag the cursor until the rectangle
is the size and shape you want.
Release the mouse button.

To make a **square**, hold down
the Shift key while you click and drag.

To make an **irregular polygon**,
select the Polygon Tool.
Click and drag to make each side of the polygon.
Double-click when you have finished.

To make a **regular polygon**, select the Regular Polygon Tool.

Click: Edit, then click: Polygon Sides...

Type in the number of sides you want. Click: OK

5. To **colour a polygon**, click the polygon to select it.

Click the Fill formatting button:

Click the Color palette button: , then select a colour.

6. Use Steps 4 and 5 to create two polygons you think are congruent.
Colour one polygon red and the other one blue.

7. To **move a polygon**, put the cursor inside the polygon.
Click and hold down the mouse button.
Drag the polygon to where you want it.
Release the mouse button.

8. To **flip a polygon**, select the polygon.

   Click: 

   then click: 

   or 

9. To check if the **polygons are congruent**:
   If you need to, flip the polygons so they have the same orientation.
   Select the red polygon.

   Click: 

   Then click: 

   Carefully drag the red polygon on top of the blue polygon.
   If you see any of the blue polygon, the polygons are not congruent.

   If you cannot see any of the blue polygon, select the red polygon.

   Click: 

   Then click: 

   If you see any of the red polygon, the polygons are not congruent.

10. Repeat Steps 6 to 9 several times.
    Use a different pair of polygons each time.

11. Save your polygons.

    Click: 

, then click: 

    Name your file. Click: 

12. Print your polygons.

    Click: 

, then click: 

    Click: 

Reflect

How can you use a computer to check if polygons are congruent?
Use pictures and words to explain.

LESSON 4

Line Symmetry

A line of symmetry is a mirror line.
It divides a figure into 2 congruent parts.
How many lines of symmetry does each Pattern Block have?

Explore

You will need grid paper, Pattern Blocks, and a Mira.

- Fold the grid paper in half.
  Use Pattern Blocks.
  Make a design on one side of the fold line.
  Your design must touch the fold line.
  Trace around your design.
  Do not draw on the fold line.
- Open the paper.
  Make a mirror image of the design on the other side of the fold line.
  Trace around the mirror image.
  Remove the Pattern Blocks.
- Find any lines of symmetry on the figure.

Show *and* Share

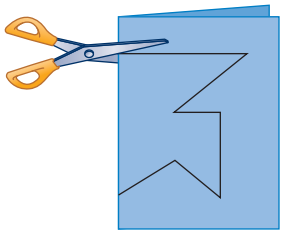
Show your work to another pair of students.
How did you find the lines of symmetry?
What did you notice about the fold line?

Connect

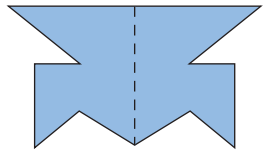
Here is one way to make a **symmetrical** figure.

A symmetrical figure has one or more lines of symmetry.

➤ Fold a piece of paper.
Draw a figure.
Use the fold line as one side of the figure.
Cut out the figure.



➤ Unfold the paper.
The fold line is a line of symmetry.



Practice

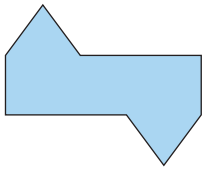
1. Trace the figures that have line symmetry.
Draw the lines of symmetry.

a)

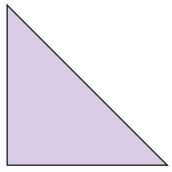
b)

c)

d)



e)

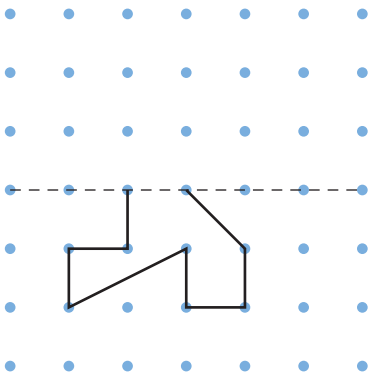


f)

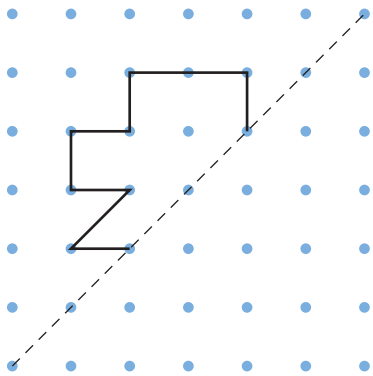
2. Choose a figure in question 1 that has line symmetry.
You will cut out this figure from folded paper.
Fold a piece of paper in half.
Use scissors. Cut out a figure so that when the paper is unfolded, it matches the figure you chose in question 1.

3. One-half of a symmetrical figure is shown.
The broken line is a line of symmetry.
Copy the figure and the line of symmetry on dot paper.
Complete the figure.

a)

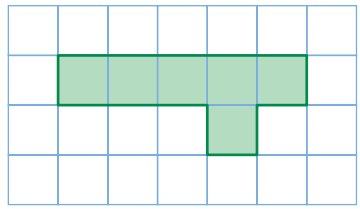


b)



4. Use a geoboard and geobands.
Divide the geoboard in half with a mirror line.
Make a figure on one-half of the geoboard.
Make its mirror image on the other half,
so the two figures form a new figure.
Is the new figure symmetrical? How do you know?

5. This figure does not have a line of symmetry.
Copy the figure on grid paper.
Add a square to it so it has a line of symmetry.
How many different ways can you do this?
Record each way on grid paper.



6. Fold a piece of paper in half.
Cut the paper to make each figure
when the paper is unfolded.

a) An isosceles triangle

b) A pentagon

Tell about the figures you created.

Reflect

How can you construct a figure that has
line symmetry?
Use pictures, numbers, or words to explain.

Numbers Every Day

Number Strategies

Order the numbers in each set from least to greatest.

- 5.37, 7.35, 3.75, 5.73, 7.53
- 0.11, 0.51, 0.15, 0.09, 1.01
- 4.88, 3.98, 1.78, 8.14, 9.38

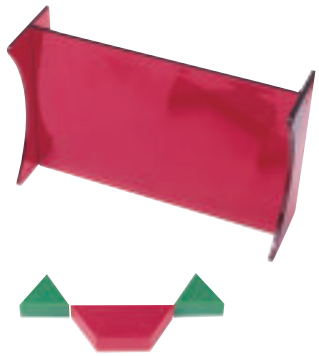
LESSON 5

Strategies Toolkit

Explore

You will need Pattern Blocks and a Mira.

Choose 3 Pattern Blocks, 2 the same and 1 different.
Arrange the 3 blocks to make a figure with exactly 1 line of symmetry.
Each block must touch at least one other block.
Trace the figure.
Draw a dotted line to show the line of symmetry.



Show *and* Share

Describe the strategy you used to solve the problem.
Could you make more than one figure? Explain.

Connect

You will need pentominoes, 2-cm grid paper, and a Mira.
Choose 2 different pentominoes.
Arrange the pentominoes to create a figure with exactly 1 line of symmetry.
Trace the figure and show the line of symmetry.

What do you know?

- Use 2 different pentominoes.
- Arrange the pentominoes to make a figure.
- The figure must have exactly 1 line of symmetry.

Think of a strategy to help you solve this problem.

- You can use **guess and check** to find a figure with exactly 1 line of symmetry.

Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.

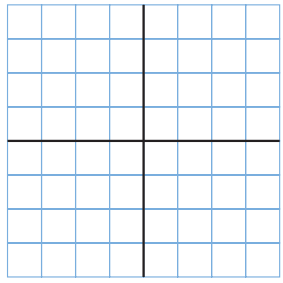
Arrange the pentominoes to make a figure.
Use a Mira to check for lines of symmetry.
If the figure has no lines of symmetry
or more than one line of symmetry,
try a different arrangement to make a new figure.

Check your work.
Does your figure have exactly 1 line of symmetry?
How do you know?

Practice

Choose one of the **Strategies**

1. Draw mirror lines to divide a piece of grid paper in 4 congruent sections.
Draw Figure A in one section.
Reflect Figure A in one of the mirror lines.
Label the image B.
Reflect Image B in the other mirror line.
Label the image C.
Describe a transformation that would move Figure A directly onto Image C.
How many different transformations can you find?



2. Repeat question 1.
This time divide the paper in 3 sections.

Reflect

How does guess and check help you solve a problem?
Use pictures and words to explain.

Fashion Designer

Fashion designers create exclusive and ready-to-wear clothing, costumes, and accessories (handbags and scarves). Successful designers hold shows in cities like Toronto, Vancouver, Montreal, Paris, and Rome. Careers in fashion range from high-profile designers to illustrators, sewers, and pattern-drafters.

The fashion designer works in both two dimensions (drawings of concepts and designs and, later, patterns) and three dimensions (actual fabrics, models, and "forms"), and must convert between them. Reflections (left and right side), translations, rotations, and tiling, as well as symmetric and asymmetric designs, have all been used to make clothes more stylish and attractive.

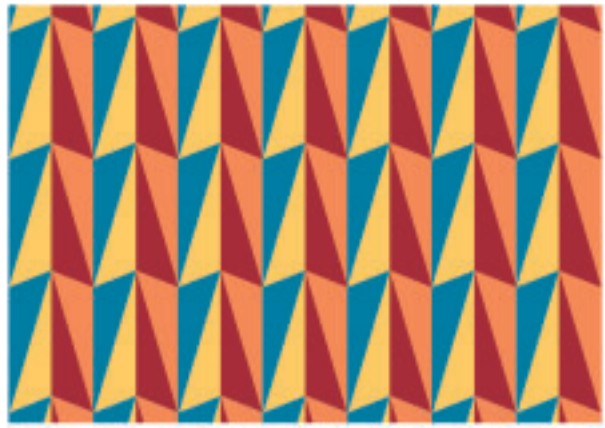
When a ready-to-wear design is prepared for mass production, computerized pattern-making technology is used. Patterns are created for cutting the material so that the least amount of fabric is wasted and costs are kept down.

LESSON 6

Exploring Tiling

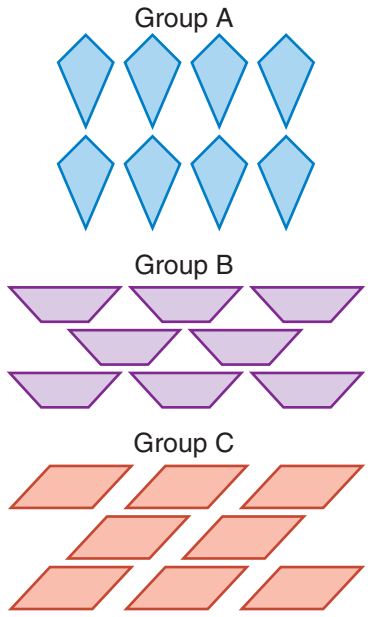
The design on this fabric is an area pattern.
There are no gaps between the figures.
None of the figures overlap.

Choose one figure to be the start figure.
Describe each other figure as an image of
the start figure after a transformation.



Explore

You will need scissors and dot paper.
Your teacher will give you a large copy of
these quadrilaterals.



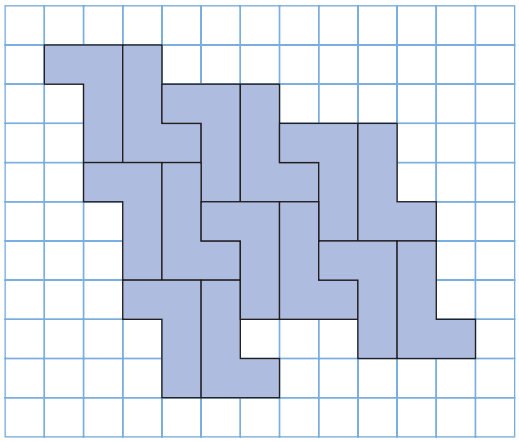
- Choose one group of quadrilaterals.
  Use scissors to cut out the quadrilaterals.
  Are the quadrilaterals in each group congruent?
  How do you know?
- Use the quadrilaterals you chose
  to create an area pattern with no gaps
  or overlaps.
  Record your pattern on dot paper.
  Describe your pattern.
- Repeat for a different group of quadrilaterals.

Show *and* Share

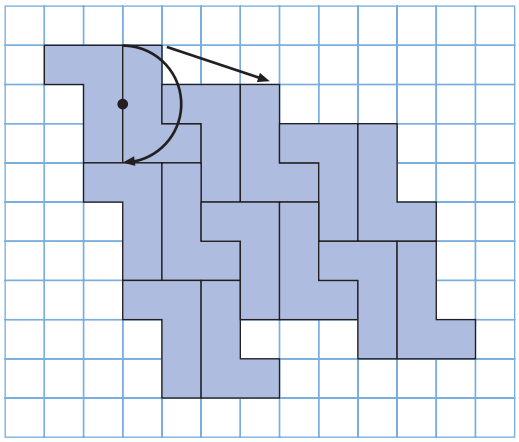
Share your patterns with another pair of students.
How did you flip, turn, or slide the quadrilaterals to make
each pattern? Explain.

Connect

A **tiling pattern** covers a surface with figures.
There are no gaps or overlaps.

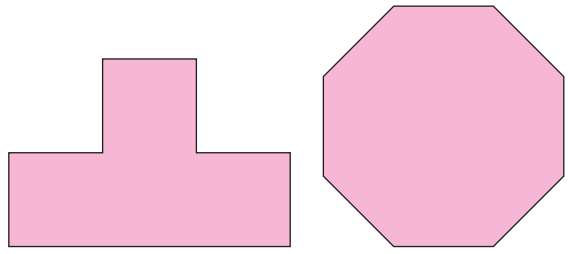


The hexagon was rotated a $\frac{1}{2}$ turn, about the dot shown.
The figure formed by a hexagon and its rotation image was translated 3 squares right and 1 square down.
To get the second row, the figure was translated 1 square right and 4 squares down.

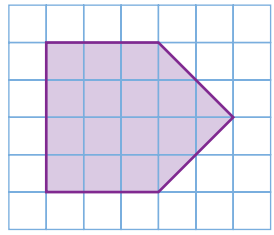


Practice

1. Use tracing paper.
Trace several copies of each octagon.
Then cut them out.
Does a regular octagon tessellate?
Does an irregular octagon tessellate?
Explain.

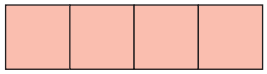


2. Copy this pentagon on grid paper.
How many different tiling patterns
can you make using this pentagon?
Describe the transformations you could use
to make each pattern.

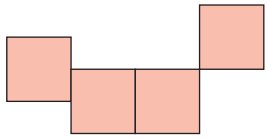


3. You will need grid paper and
4 congruent squares.
A tetromino is made with 4 congruent squares.
Each square must align with at least one other square
along one edge.

This is a tetromino.



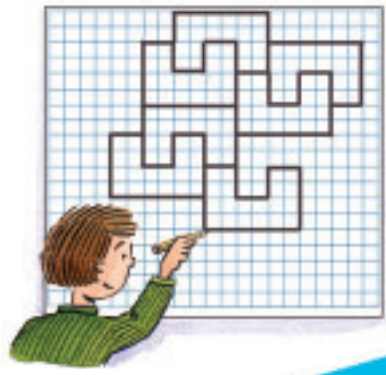
This is not a tetromino.



a) How many different tetrominoes can you find?
Record each tetromino on grid paper.

b) Select one of your tetrominoes.
Does it tessellate? Explain.

4. Use grid paper.
Draw a figure.
Transform the figure to create a tiling pattern.
Describe the transformations you used.



5. Create a pentagon you think will tessellate.
Trace several copies of the figure.
Then cut them out.
Does the figure tessellate?
Tell why or why not.

Reflect

How can you use transformations
to describe a tessellation?
Use pictures and words to explain.

Numbers Every Day

Number Strategies

Write each number as a decimal.

- thirty and fifty-three hundredths
- thirty-five and three-tenths
- twenty-two hundredths
- twenty and two-hundredths
- twenty-two and two-tenths

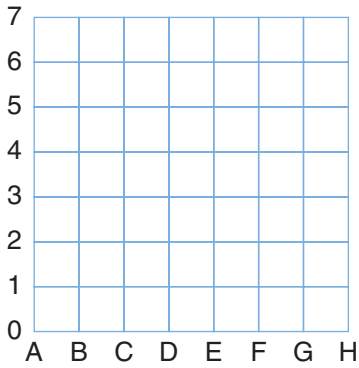
Unit 7 Show What You Know

LESSON

1

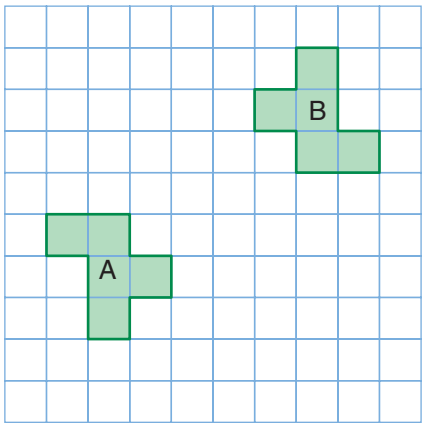
1. Use grid paper. Copy this grid.
Mark a point at each pair of coordinates.
Connect the points in each group, in order.
What did you draw?

- B7, D7, D3, B3
- B5, D5
- E7, E3, G3, G5, E5



2

2. Describe a transformation that would move Figure A so it coincides with Figure B.

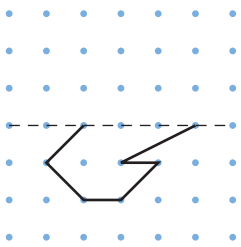


2
3

3. Use dot paper. Draw 2 congruent figures.
Describe the transformations that would move one figure so it coincides with the other.

4

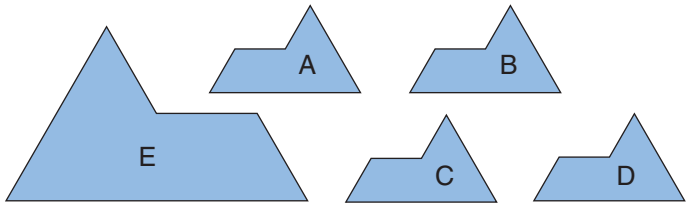
4. One-half of a symmetrical figure is shown.
The broken line is a line of symmetry.
Copy the figure and the line of symmetry on dot paper.
Complete the figure.



LESSON

2 3 6

5. You will need a large copy of these figures. Cut out the figures.

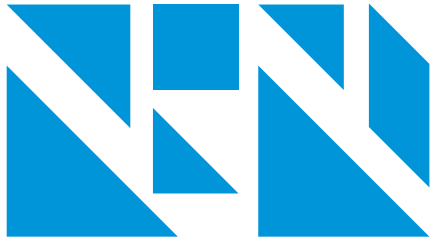


a) Which figures are congruent? How do you know?

b) Arrange Figures A, B, C, and D to make a figure congruent to E. Record your arrangement.

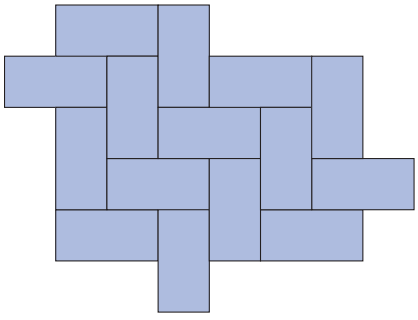
3

6. There are 7 tans in a tangram. Are any tans congruent? How do you know?



2 6

7. Select one figure. Explain how each other figure is the image after a transformation.



8. You will need triangular grid paper and 6 green Pattern Blocks. Arrange all 6 triangles to make a figure. At least one side of each triangle must align with one side of another triangle.

a) How many different figures can you make? Record each figure on grid paper.

b) Select one of your figures. Does it tessellate? Explain.

UNIT 7 Learning Goals

- ☑ describe position on a grid
- ☑ apply translations, reflections, and rotations
- ☑ pose and solve problems with transformations
- ☑ identify congruent figures
- ☑ construct figures with one line of symmetry
- ☑ explore tiling patterns and tessellations

Unit Problem

Geometry in Art

You will need:
- Bristol board
- coloured paper
- a 10-cm square cut from 1-cm grid paper
- scissors

Part 1

Make a pattern for a tile that tessellates.

- Draw a figure on the grid paper square.
  One side of the figure must coincide with one side of the square.

- Cut out the figure.
- Slide the figure across the square and tape it to the opposite side. The edges must line up with no gaps or overlaps.
- Continue to cut figures and tape them to the opposite side until your pattern is the way you want it.

Part 2

Make a tessellation.

- Use your pattern to cut tiles from coloured paper.
- Glue the tiles onto Bristol board.

Part 3

Write about your tessellation.

- Does your tessellation have line symmetry?
  How do you know?
- How could you use transformations to make the tessellation?
  Describe as many ways as you can find.

Check List

Your work should show

- ☑ a pattern for a tile that tessellates
- ☑ a tesselation of coloured tiles on Bristol board
- ☑ a description of whether or not your tessellation has line symmetry
- ☑ an explanation of how you could use transformations to make the tessellation

Reflect on the Unit

What do you know about transformations and congruent figures?
Use pictures and words to explain.

Cross Strand Investigation

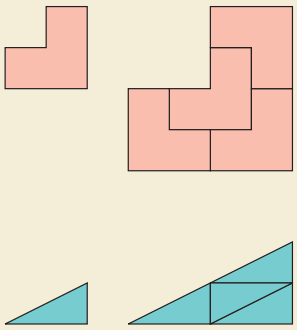
Rep-Tiles

You will need Pattern Blocks.

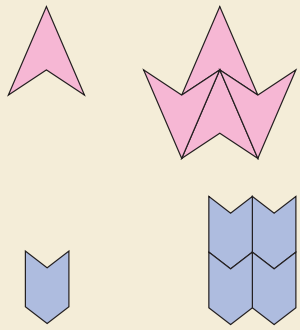
Part 1

A **rep-tile** is a polygon that can be copied and arranged to form a larger, similar polygon.

These are rep-tiles:



These are not rep-tiles:



➤ Which Pattern Blocks are rep-tiles?
How did you find out?

Part 2

Choose a block that is a rep-tile.
Do not use orange or green blocks.
Build a growing pattern.
Record the pattern.

➤ Choose one Pattern Block that is a rep-tile.
This is Frame 1.

➤ Now take several of the same type of block.
Arrange the blocks to form a similar polygon.
This is Frame 2.
Continue to arrange blocks to make larger similar polygons.
The next largest polygon is Frame 3.

- Suppose the side length of the green Pattern Block is 1 unit.
  Find the perimeter of each figure.
- Suppose the area of the green Pattern Block is 1 square unit.
  Find the area of each frame.
  Copy and complete the table.

Frame	Number of Blocks	Perimeter	Area
1	1		
2			

Part 3

- What patterns can you find in the table?
- How many blocks would you need to build Frame 7?
  How do you know?
- Predict the area and the perimeter of Frame 9.
  How did you make your prediction?

Display Your Work

Record your work.
Describe the patterns you found.

Take It Further

Draw a large polygon you think is a rep-tile.
Trace several copies.
Cut them out.
Try to arrange the copies to make a larger similar polygon.
If your polygon is a rep-tile, explain why it works.
If it is not, describe how you could change it to make it work.